The Optimal Number Model for All-Electric Cars Based on M/M/s/N Queuing Model---Take the United States of America as an example

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Abstract: Based on the analysis of the demand of a region and based on the queuing theory, this paper establishes a model to determine the optimal number of charging stations in a region. The purpose is to meet the charging requirements of users without generating waste, so as to maximize the economic and social benefits. This paper selects the data of the US Tesla company and the relevant parameter of the Tesla's charging stations as a reference. Because the United States has a vast territory and people there have a high willingness to drive a car. What's more, the popularity of electric vehicles is relatively high. At the same time, Tesla's products are relatively complete in supporting facilities, which takes a certain reference role in the development of China's future all-electric vehicles.

1. Assumptions

The car charging piles located inside the city are evenly distributed and the car has the same probability to each charging piles.

The average charge of the car reaching the charging pile is 20%.

Cars located inside the city reach the charging post obey the Poisson stream.

Electric car fully charged to meet the needs of the city five days, driving 3.5 days in rural areas, rural driving demand for two days.

There is only one charging post per charging station.

2. Notation

Symbol	*Description*
A	The arrival strength
λ	The parameter which the time interval when the customer reaches the system follows
N	the maximum number of customers allowed in a charging station
s	The number of charging piles
θ	The parameter which the possibility for the customers to wait for charging patiently follows
μ	The parameter which the time for the customers to accomplish charging patiently follows

3. Model development

Queuing models are often used to determine the number of configurations in a service establishment [1].We call this behavior a "stop" for our customers. Whether it is because of the limited capacity of the system, or because the customer is impatient or stopping, the customer leaves the system without service, it is negative for the place of charging in terms of economic or social benefits. In this paper, we use the limited capacity M / M / s / N queuing model with impatience and stopping behavior of customers to model the charging service system of charging place, and establish the benefit function of charging service system. Taking the maximum benefit function as the target function, the number of charging stations in charging place is optimized.

Charging service system has a total of s charging piles, and the system capacity of N. When the customer arrives at the charging place and finds that there is a spare charging station, the customer directly selects the spare charging pile to charge. If the charging station has no free charging piles, customers can wait for charging service in the waiting area of the system. A total of $(N-s)$ waiting area waiting seats, but also can not join the waiting queue and stop leaving the system. When the customer arrives at the charging place, if there are already N customers in the system, the customer leaves the system directly. In addition, customers in the waiting area may be impatient to leave due to long waiting times. The above charging service system uses a limited capacity M / M / s / N queuing system modeling with impatience and stopping behavior for the customer.

Arrival process: The customer arrives at the system according to the Poisson flow, and the arrival strength is A, that is, the time interval when the customer reaches the system follows the exponential distribution with the parameter λ.

Queuing rules: charging service system capacity is limited, when the number of customers into the system reaches N, the system no longer accept later customers. The system has a total of s charging stations, the system can accommodate up to a car charging. When a customer entering the system finds that s charging stations are full, it leaves the system with a probabilistic halt. When the number of seats in the waiting area is $(N-s)$, when the car is fully charged and leaves the system, the first car on the waiting area will be charged with the vacated charging station, that is, the service will be carried out according to the principle of first come first served.

Customers in the waiting area will be impatient to leave due to long wait times, assuming the customer waited patiently for time to follow the exponential distribution of parameter θ. Customer charging time independent of each other, and subject to the parameter μ exponential distribution.

4. System Steady-state Performance Characteristics[2]

We assume that $N(t)$ is the number of customers at the time of. Because customer arrival time interval, service time are independent of each other and are subject to exponential distribution, so the process $\{N(t), t \geq 0\}$ is the continuous time Markov chain, its state space is $\Omega = \{0,1,2,\ldots,N\}$. Obviously, $\{N(t), t \geq 0\}$ is a birth and death process, and the birth rate λ and death rate μ are shown below.

$$\lambda_i = \begin{cases} \lambda & \text{i=0,1,...,s} \\ \lambda_q & \text{i=s+1.s+2,...,N} \end{cases} \tag{1}$$

$$\mu_i = \begin{cases} i\mu & \text{i=1,2,...,s} \\ s\mu+(i-s)\theta & \text{i=s+1,s+2,...,N} \end{cases} \tag{2}$$

We assume that $p_n = \lim_{t\to\infty} p\{N(t)=n\}, n=0,1,\ldots,N$, so the steady-state equation of the system is

$$\lambda p_0 = \mu p_1, \tag{3}$$

$$(\lambda + n\mu) p_n = \lambda p_{n-1} + (n+1)\mu p_{n+1}, 1 \leq n \leq s-1 \tag{4}$$

$$(\lambda q + s\mu) p_c = \lambda p_{s-1} + (s\mu + \theta) p_{s+1}, \tag{5}$$

$$\{\lambda q + [s\mu + (n-s)\theta]\} p_n = \lambda q p_{n-1} + [s\mu + (n+1-s)\theta] p_{s+1}, s+1 \leq \text{n} \leq \text{N-1} \tag{6}$$

$$[s\mu + (N-s)\theta] p_N = \lambda q p_{N-1} \tag{7}$$

Solve the balance equations, we get:

$$p_n = \begin{cases} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n p_0, & 0 \le n \le s, \\ \frac{q^{n-s}\lambda^n}{s!\mu^s \prod_{j=1}^{n-s}(s\mu - j\theta)} p_0, & s+1 \le n \le N \end{cases} \tag{8}$$

$$\sum_{n=0}^{N} p_n = 1 \tag{9}$$

Then due to normality $\sum_{n=0}^{N} p_n = 1$, and (8) and (9),we can get the System steady-state performance index.

Probability of charging station charging station all free:

$$p_0 = \left[\sum_{n=0}^{s} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \sum_{n=s+1}^{N} \frac{q^{n-s}\lambda^n}{s!\mu^s \prod_{j=1}^{n-s}(s\mu + j\theta)} \right]^{-1} \tag{10}$$

The average number of vehicles to stay in the charging place:

$$L_s = \sum_{n=0}^{N} np_n = \left[\sum_{n=1}^{s} \frac{1}{(n-1)!}\left(\frac{\lambda}{\mu}\right)^n + \sum_{n=s+1}^{N} \frac{nq^{n-s}\lambda^n}{s!\mu^s \prod_{j=1}^{n-s}(s\mu + j\theta)} \right] p_0 \tag{11}$$

The average utilization rate of charging stations:

$$Q = \sum_{n=0}^{s} \frac{n}{s} p_n + \sum_{n=s+1}^{N} p_n = \left[\sum_{n=1}^{s} \frac{1}{s(n-1)!}\left(\frac{\lambda}{\mu}\right)^n + \sum_{n=s+1}^{N} \frac{q^{n-s}\lambda^n}{s!u^s \prod_{j=1}^{n-s}(s\mu + j\theta)} \right] p_0 \tag{12}$$

The average number of vehicles waiting to be charged at the charging station:

$$L_q = L_s - sQ \tag{13}$$

Charging service system no longer accepts the customer's probability:

$$P_f = p_N = \frac{q^{N-s}\lambda^N}{s!\mu^s \prod_{j=1}^{N-s}(s\mu + j\theta)} p_0 \tag{14}$$

Probability of charging service system’s loss of customer due to stop service:

$$P_B = (1-q)\sum_{n=s}^{N-1} P_n = (1-q)\left[\frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s + \sum_{n=s+1}^{N-1} \frac{q^{n-s}}{s!\mu^s \prod_{j=1}^{n-s}(s\mu + j\theta)} \right] p_0 \tag{15}$$

Charging service system’s total rate of customer loss:

$$P_L = \frac{\lambda P_f + \lambda P_B + L_q\theta}{\lambda} = P_f + P_B + \frac{L_q\theta}{\lambda} \tag{16}$$

5. Quantity optimization design

In the case of charging stations, the ideal situation is that the charging posts of charging stations can be fully utilized, and the customers who want to charge them do not need to wait for a long time. If the charging station is too few charging piles, will easily lead customers to stop or left waiting impatient to leave too long, resulting in some charging to the economic loss.

Therefore, we look for the optimal charging pile configuration number s^* from the viewpoint of maximizing the total benefit of charging service system. Let $P_L(s)$ be the total customer loss rate for charging service system charging pile configuration number s. Suppose α is the unit customer receives the charge service to the charging place to bring the benefit. β is the unit of customer service without leaving the system to the charging site caused the loss. γ is the unit of service charge per unit of service time, which consists of electricity, equipment depreciation costs, maintenance costs and operating costs and so on. The benefits and losses here need to be set from the perspectives of both economic and social benefits.

We define the loss function of the charging service system as:

$$z = \alpha s + \beta L \tag{17}$$

$$L = \frac{P_0\left(\frac{\alpha}{\beta}\right)^{s+1}}{(s-1)!\left(s-\frac{\alpha}{\beta}\right)^2} + \frac{\alpha}{\beta} \tag{18}$$

$$p_0 = \left[\sum_{n=0}^{s-1}\frac{\left(\frac{\alpha}{\beta}\right)^n}{n!} + \frac{\left(\frac{\alpha}{\beta}\right)^s}{(s-1)!\left(s-\frac{\alpha}{\beta}\right)}\right]^{-1} \tag{19}$$

The only variable is the number of charging piles, so we can think of z as a function of s. Denote $z = z(s)$ and find s^* that minimizes $z(s)$. Because s takes only integers, $z(s)$ is not a continuous function, so the marginal analysis method. According to $z(s^*)$ should be the smallest features,

$z(s*) \le z(s*-1) \qquad z(s*) \le z(s*+1)$

So

$\alpha s*+ \beta L(s*) \le \alpha(s*-1) + \beta L(s*-1)$

$\alpha s*+ \beta L(s*) \le \alpha(s*+1) + \beta L(s*+1)$

Get simplified

$L(s*) - L(s*+1) \le \frac{\alpha}{\beta} \le L(s*-1) - L(s*)$

We can find the optimal solution s*.

References

[1] Zheng Meng Lei. Based on queuing theory of electric vehicle charging station scale optimization. Southwest Jiaotong University, 2016.

[2] He zhanyong. Electric vehicle charging planning method and operation mode. Beijing Jiaotong University, 2015.